Bounds on CPT and Lorentz Violation from Experiments with Kaons*

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IUHET 411, July 1999

Abstract

Possible signals for indirect CPT violation arising in experiments with neutral kaons are considered in the context of a general CPT- and Lorentz-violating standard-model extension. Certain CPT observables can depend on the meson momentum and exhibit sidereal variations in time. Any leading-order CPT violation would be controlled by four parameters that can be separately constrained in appropriate experiments. Recent experiments bound certain combinations of these parameters at the level of about 10^{-20} GeV.

Experiments using neutral-meson oscillations can place constraints of remarkable precision on possible violations of CPT invariance. For kaons, recent results [1, 2, 3] bound the CPT figure of merit $r_K \equiv |m_K - m_{\overline{K}}|/m_K$ to less than a part in 10^{18}. Other experiments [4, 5, 6] are expected to improve this bound in the near future. Experiments with neutral-B mesons [7, 8] have also placed high-precision constraints on possible CPT violation, and the B and charm factories should produce additional bounds on the heavy neutral-meson systems.

A purely phenomenological treatment of possible CPT violation in the kaon system has been known for some time [9]. In this approach, a complex phenomenological parameter δ_K allowing for indirect CPT violation is introduced in the standard relationships between the physical meson states and the strong-interaction eigenstates. No information about δ_K itself can be obtained within this framework. However, over the past ten years a plausible theoretical framework allowing the possibility of CPT violation has been developed. It involves the notion of spontaneous breaking of CPT and Lorentz symmetry in a fundamental theory [10], perhaps arising at the Planck scale from effects in a quantum theory of gravity or string theory,

*Invited talk at KAON '99, Chicago, Illinois, June 1999

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and it is compatible both with established quantum field theory and with present experimental constraints. At low energies, a general CPT- and Lorentz-violating standard-model extension emerges that preserves gauge invariance and renormalizability [11, 12] and that provides an underlying basis for the phenomenology of CPT violation in the kaon system. The resulting situation is comparable to that for conventional CP violation, where the nonzero value of the phenomenological parameter ϵ_K for T violation in the kaon system can in principle be calculated from the usual standard model of particle physics [13, 14].

In this talk, the primary interest is in the application of the standard-model extension to CPT tests with kaons. However, the standard-model extension also provides a quantitative microscopic framework for CPT and Lorentz violation that can be used to evaluate and compare a wide variety of other experiments [15]. These include tests with heavy neutral-meson systems [7, 8, 11, 16, 17], studies of fermions in Penning traps [18, 19, 20, 21, 22], constraints on photon birefringence and radiative QED effects [12, 23, 24, 25], hydrogen and antihydrogen spectroscopy [26, 27], clock-comparison experiments [28, 29], measurements of muon properties [30], cosmic-ray and neutrino tests [31], and baryogenesis [32].

Developing a plausible theoretical framework for CPT violation without radical revisions of established quantum field theory is a difficult proposition [15, 33]. It is therefore perhaps to be expected that in the context of the standard-model extension the parameter δ_K displays features previously unexpected, including dependence on momentum magnitude and orientation. The implications include, for instance, time variations of the measured value of δ_K with periodicity of one sidereal (not solar) day [17].

First, consider some general theoretical features relevant for oscillations in any neutral-meson system. Denote generically the strong-interaction eigenstate by P^0, where P^0 is one of K^0, D^0, B_d^0, B_s^0, and denote the opposite-flavor antiparticle by $\overline{P^0}$. Then a neutral-meson state is a linear combination of the Schrödinger wave function for P^0 and $\overline{P^0}$. The time evolution of the associated two-component object Ψ state is given [9] in terms of a 2×2 effective hamiltonian Λ by $i\partial_t\Psi = \Lambda\Psi$. The physical propagating states are the eigenstates P_S and P_L of Λ. They have eigenvalues $\lambda_S \equiv m_S - \frac{1}{2}i\gamma_S$ and $\lambda_L \equiv m_L - \frac{1}{2}i\gamma_L$, respectively, where m_S, m_L are the propagating masses and γ_S, γ_L are the associated decay rates. Flavor oscillations between P^0 and $\overline{P^0}$ are controlled by the off-diagonal components of Λ, while indirect CPT violation [34] occurs if and only if the diagonal elements of Λ have a nonzero difference $\Lambda_{11} - \Lambda_{22} \neq 0$. Writing Λ as $\Lambda \equiv M - \frac{1}{2}i\Gamma$, where M and Γ are hermitian, the condition for CPT violation becomes $\Delta M - \frac{1}{2}i\Delta\Gamma \neq 0$, where $\Delta M \equiv M_{11} - M_{22}$ and $\Delta\Gamma \equiv \Gamma_{11} - \Gamma_{22}$.

A perturbative calculation in the general standard-model extension pro-

vides the dominant CPT-violating contributions to Λ [10]. It turns out that the hermiticity of the perturbing hamiltonian enforces $\Delta\Gamma = 0$ at leading order. The leading-order signal therefore arises in the difference ΔM, and so the standard figure of merit

$$r_P \equiv \frac{|m_P - m_{\overline{P}}|}{m_P} = \frac{|\Delta M|}{m_P} \tag{1}$$

provides a complete description of the magnitude of the dominant CPT-violating effects. An explicit expression for ΔM in terms of quantities in the standard-model extension is known [11, 17]. For several reasons, its form turns out to be relatively simple,

$$\Delta M \approx \beta^\mu \Delta a_\mu \quad . \tag{2}$$

Here, $\beta^\mu = \gamma(1, \vec{\beta})$ is the four-velocity of the meson state in the observer frame and Δa_μ is a combination of CPT- and Lorentz-violating coupling constants for the two valence quarks in the P^0 meson. Note that the oscillation experiments considered here provide the only known sensitivity to Δa_μ. Note also that the velocity dependence and the corresponding momentum dependence of ΔM is compatible with the anticipated substantial modifications to standard physics if the CPT theorem is violated.

The experimental implications of momentum dependence in observables for CPT violation are substantial. Effects can be classified according to whether they arise primarily from a dependence on the magnitude of the boost or from the variation with its direction [17]. The dependence on momentum magnitude implies the possibility of increasing the CPT reach by changing the meson boost and even the possibility of increasing sensitivity by restricting attention to a momentum subrange in a given dataset. The dependence on momentum direction implies variation of observables with the beam direction for collimated mesons, variation with the meson angular distribution for other situations, and sidereal effects arising from the rotation of the Earth relative to the constant 3-vector $\Delta\vec{a}$. In actual experiments the momentum and angular dependences are frequently used to determine detector properties and experimental systematics, so there is a definite risk of cancelling or averaging away CPT-violating effects. However, the detection of a momentum dependence in observables would be a unique feature of CPT violation. There are also new possibilities for data analysis. For instance, measurements of an observable can be binned according to sidereal time to search for possible time variations as the Earth rotates.

The above discussion holds for any neutral-meson system. For definiteness, the remainder of this talk considers the special case of kaons. The parameter δ_K, which is effectively a phase-independent quantity, can

be defined through the relationship between the eigenstates of the strong interaction and those of the effective hamiltonian:

$$
\begin{aligned}
|K_S\rangle &= \frac{(1+\epsilon_K+\delta_K)|K^0\rangle + (1-\epsilon_K-\delta_K)|\overline{K^0}\rangle}{\sqrt{2(1+|\epsilon_K+\delta_K|^2)}} \quad , \\
|K_L\rangle &= \frac{(1+\epsilon_K-\delta_K)|K^0\rangle - (1-\epsilon_K+\delta_K)|\overline{K^0}\rangle}{\sqrt{2(1+|\epsilon_K-\delta_K|^2)}} \quad .
\end{aligned} \tag{3}
$$

Assuming that all CP violation is small, δ_K is in general given as

$$
\delta_K \approx \Delta\Lambda / 2\Delta\lambda \quad , \tag{4}
$$

where $\Delta\lambda \equiv \lambda_S - \lambda_L$ is the eigenvalue difference of Λ. In terms of the mass and decay-rate differences $\Delta m \equiv m_L - m_S$ and $\Delta\gamma \equiv \gamma_S - \gamma_L$, it follows that $\Delta\lambda = -\Delta m - \frac{1}{2} i \Delta\gamma = -i\Delta m e^{-i\hat{\phi}} / \sin\hat{\phi}$, where $\hat{\phi} \equiv \tan^{-1}(2\Delta m/\Delta\gamma)$.

In the context of the standard-model extension, the above expressions show that a meson with velocity $\vec{\beta}$ and corresponding boost factor γ displays CPT-violating effects given by

$$
\delta_K \approx i \sin\hat{\phi}\; e^{i\hat{\phi}} \gamma (\Delta a_0 - \vec{\beta}\cdot\Delta\vec{a})/\Delta m \quad . \tag{5}
$$

The conventional figure of merit r_K becomes

$$
\begin{aligned}
r_K &\equiv \frac{|m_K - m_{\overline{K}}|}{m_K} \approx \frac{2\Delta m}{m_K \sin\hat{\phi}} |\delta_K| \\
&\approx \frac{|\beta^\mu \Delta a_\mu|}{m_K} \quad .
\end{aligned} \tag{6}
$$

After substitution for the known experimental values [35] for Δm, m_K, and $\sin\hat{\phi}$, this gives

$$
r_K \simeq 2\times 10^{-14} |\delta_K| \simeq 2 \left| \beta^\mu \frac{\Delta a_\mu}{1\ \mathrm{GeV}} \right| \quad . \tag{7}
$$

A constraint on $|\delta_K|$ of about 10^{-4} corresponds to a limit on $|\beta^\mu \Delta a_\mu|$ of about 10^{-18} GeV.

The dependence of the eigenfunctions and eigenvalues of Λ on M_{11} and M_{22} raises the possibility of leading-order momentum dependence in the parameter ϵ_K, in the masses and decay rates m_S, m_L, γ_S, γ_L, and in various associated quantities such as Δm, $\Delta\gamma$, $\hat{\phi}$. However, this possible dependence is in fact absent because the CPT-violating contribution from M_{22} is the negative of that from M_{11}, and only δ_K is sensitive to ΔM at leading order. Thus, for example, the usual parameter ϵ_K for indirect T violation is independent of momentum in the present framework [36].

The expressions obtained above can be viewed as defined in the laboratory frame. To exhibit the time dependence of δ_K arising from the rotation of the Earth, a different and nonrotating frame is useful [29]. A basis $(\hat{X}, \hat{Y}, \hat{Z})$ for this frame can be introduced in terms of celestial equatorial coordinates. The $\hat{Z}$ axis is defined as the rotation axis of the Earth, while $\hat{X}$ has declination and right ascension 0° and $\hat{Y}$ has declination 0° and right ascension 90°. This provides a right-handed orthonormal basis that is independent of any particular experiment. Denote the spatial basis in the laboratory frame as $(\hat{x}, \hat{y}, \hat{z})$, where $\hat{z}$ and $\hat{Z}$ differ by a nonzero angle given by $\cos\chi = \hat{z} \cdot \hat{Z}$. Then, $\hat{z}$ precesses about $\hat{Z}$ with the Earth's sidereal frequency Ω. A convenient choice of $\hat{z}$ axis is often along the beam direction. If the origin of time $t = 0$ is taken such that $\hat{z}(t = 0)$ is in the first quadrant of the $\hat{X}$-$\hat{Z}$ plane and if $\hat{x}$ is defined perpendicular to $\hat{z}$ and lies in the $\hat{z}$-$\hat{Z}$ plane for all t, then a right-handed orthonormal basis can be completed with $\hat{y} := \hat{z} \times \hat{x}$. It follows that $\hat{y}$ lies in the plane of the Earth's equator and is perpendicular to $\hat{Z}$ at all times. Disregarding relativistic effects due to the rotation of the Earth, a nonrelativistic transformation (given by Eq. (16) of Ref. [29]) provides the conversion between the two bases.

Using the above results, one can obtain in the nonrotating frame an expression for the parameter δ_K in the general case of a kaon with three-velocity $\vec{\beta} = \beta(\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Here, θ and ϕ are standard spherical polar coordinates specified in the laboratory frame about the $\hat{z}$ axis. If $\hat{z}$ coincides with the beam axis, the spherical polar coordinates can be taken as the usual polar coordinates for a detector. One finds

$$\begin{aligned}
\delta_K(\vec{p}, t) \quad = \quad & \frac{i \sin\hat{\phi}\ e^{i\hat{\phi}}}{\Delta m} \gamma(\vec{p}) \Big[\Delta a_0 + \beta(\vec{p}) \Delta a_Z (\cos\theta\cos\chi - \sin\theta\cos\phi\sin\chi) \\
& \qquad + \beta(\vec{p}) \left(-\Delta a_X \sin\theta\sin\phi \right. \\
& \qquad\qquad \left. + \Delta a_Y (\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi) \right) \sin\Omega t \\
& \qquad + \beta(\vec{p}) \left(\Delta a_X (\cos\theta\sin\chi + \sin\theta\cos\phi\cos\chi) \right. \\
& \qquad\qquad \left. + \Delta a_Y \sin\theta\sin\phi \right) \cos\Omega t \Big] \quad ,
\end{aligned} \tag{8}$$

where $\gamma(\vec{p}) = \sqrt{1 + |\vec{p}|^2/m_K^2}$ and $\beta(\vec{p}) = |\vec{p}|/m\gamma(\vec{p})$, as usual. This expression has direct implications for experiment. For example, the complex phase of δ_K is $i\exp(i\hat{\phi})$, independent of momentum and time. The real and imaginary parts of δ_K therefore exhibit the same momentum and time dependence, and so $\mathrm{Re}\,\delta_K$ and $\mathrm{Im}\,\delta_K$ scale proportionally when a meson is boosted. Another property of Eq. (8) is the variation of the CPT-violating effects with the meson boost. For example, if $\Delta a_0 = 0$ in the laboratory frame then there is no CPT violation for a meson at rest but effects appear when the meson is boosted. In contrast, for the case where $\Delta\vec{a} = 0$ in the laboratory frame, CPT violation is enhanced by the boost factor γ relative

to a meson at rest. Other implications follow from the angular dependence in Eq. (8) and from the variation of δ_K with sidereal time t. For example, under some circumstances all CPT violation can average to zero if, as usual, neither angular separation nor time binning are performed.

The momentum and time dependence given by Eq. (8) implies that the experimental setup and data-taking procedure affect the CPT reach. Space restrictions here preclude consideration of all the different classes of scenario realized in practice. Instead, attention is restricted here to a single one, typified by the E773 and KTeV experiments [1, 37]. This class of experiment, which involves highly collimated uncorrelated kaons having nontrivial momentum spectrum and large mean boost, is particularly relevant here because the KTeV collaboration announced at this conference the first constraints on the sidereal-time dependence of CPT observables in the kaon system [2]. A discussion of some issues relevant to other types of experiment can be found in Ref. [17].

The KTeV experiment involves kaons with $\beta \simeq 1$ and average boost factor $\overline{\gamma}$ of order 100. For this case, $\hat{z} \cdot \hat{Z} = \cos\chi \simeq 0.6$. In all experiments with boosted collimated kaons, Eq. (8) simplifies because the kaon three-velocity in the laboratory frame can be taken as $\vec{\beta} = (0, 0, \beta)$. The expression for δ_K becomes

$$\delta_K(\vec{p}, t) = \frac{i \sin\hat{\phi}\ e^{i\hat{\phi}}}{\Delta m}\gamma[\Delta a_0 + \beta\Delta a_Z \cos\chi + \beta \sin\chi(\Delta a_Y \sin\Omega t + \Delta a_X \cos\Omega t)]. \tag{9}$$

In this equation, each of the four components of Δa_μ has momentum dependence through the boost factor γ. However, only the coefficients of Δa_X and Δa_Y vary with sidereal time.

To gain insight into the implications of Eq. (9), consider first a conventional analysis that seeks to constrain the magnitude $|\delta_K|$ but disregards the momentum and time dependence. Assuming the experiment is performed over an extended time period, as is typically the case, the relevant quantity is the time and momentum average of Eq. (9):

$$|\overline{\delta_K}| = \frac{\sin\hat{\phi}}{\Delta m}\overline{\gamma}(\Delta a_0 + \overline{\beta}\Delta a_Z \cos\chi) \quad , \tag{10}$$

where $\overline{\beta}$ and $\overline{\gamma}$ are appropriate averages of β and γ, respectively, taken over the momentum spectrum of the data. Substitution of the experimental quantities and the current constraint on $|\delta_K|$ from this class of experiment permits the extraction of a bound on a combination of Δa_0 and Δa_Z [17]:

$$|\Delta a_0 + 0.6\Delta a_Z| \lesssim 10^{-20} \text{ GeV} \quad . \tag{11}$$

The ratio of this to the kaon mass compares favorably with the ratio of the kaon mass to the Planck scale. Note that the CPT reach of this class of

experiments is some two orders of magnitude greater than might be inferred from the bound on r_K, due to the presence of the boost factor $\overline{\gamma} \simeq 100$.

In experiments with kaon oscillations, the bounds obtained on δ_K are extracted from measurements on other observables including, for instance, the mass difference Δm, the K_S lifetime $\tau_S = 1/\gamma_S$, and the ratios η_{+-}, η_{00} of amplitudes for 2π decays. The latter are defined by

$$\begin{aligned} \eta_{+-} &\equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv |\eta_{+-}| e^{i\phi_{+-}} \approx \epsilon + \epsilon' \quad , \\ \eta_{00} &\equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv |\eta_{00}| e^{i\phi_{00}} \approx \epsilon - 2\epsilon' \quad . \end{aligned} \tag{12}$$

Adopting the Wu-Yang phase convention [38], it follows that $\epsilon \approx \epsilon_K - \delta_K$ [39, 40]. Experimentally, it is known that $|\epsilon| \simeq 2 \times 10^{-3}$ [35] and that $|\epsilon'| \simeq 6 \times 10^{-6}$ [41]. Since δ_K is bounded only to about 10^{-4} it is acceptable at present to neglect ϵ', equivalent to assuming the hierarchy $|\epsilon_K| > |\delta_K| > |\epsilon'|$. Noting that the phases of ϵ_K and δ_K differ by 90° [42] then gives

$$\begin{aligned} |\eta_{+-}| e^{i\phi_{+-}} &\approx |\eta_{00}| e^{i\phi_{00}} \approx \epsilon \approx \epsilon_K - \delta_K \\ &\approx (|\epsilon_K| + i|\delta_K|) e^{i\hat{\phi}} \quad . \end{aligned} \tag{13}$$

This implies

$$\begin{aligned} |\eta_{+-}| &\approx |\eta_{00}| \approx |\epsilon_K|(1 + O(|\delta_K/\epsilon_K|^2) \quad , \\ \phi_{+-} &\approx \phi_{00} \approx \hat{\phi} + |\delta_K/\epsilon_K| \quad , \end{aligned} \tag{14}$$

which shows that leading-order momentum and time dependences in measured quantities appear only in the phases ϕ_{+-} and ϕ_{00}. The momentum and time dependences are absent or suppressed in other observables, including $|\eta_{+-}|$, $|\eta_{00}|$, ϵ', Δm, $\hat{\phi}$, and $\tau_S = 1/\gamma_S$.

Substituting for δ_K in ϕ_{+-} and ϕ_{00}. yields expressions displaying explicitly the time and momentum dependences:

$$\begin{aligned} \phi_{+-} \approx \phi_{00} &\approx \hat{\phi} + \frac{\sin\hat{\phi}}{|\eta_{+-}|\Delta m} \gamma[\Delta a_0 + \beta \Delta a_Z \cos\chi \\ &\qquad + \beta \sin\chi (\Delta a_Y \sin\Omega t + \Delta a_X \cos\Omega t)]. \end{aligned} \tag{15}$$

Since the coefficients of each of the four components Δa_0, Δa_X, Δa_Y, Δa_Z are all distinct, this equation shows that in principle each component can be independently bounded in the class of experiments involving collimated kaons with a nontrivial momentum spectrum. Thus, binning in time and fitting to sine and cosine terms would allow independent constraints on Δa_X and Δa_Y, while a time-averaged analysis would permit the extraction of Δa_0 and Δa_Z provided the momentum spectrum includes a significant

range of $\vec{\beta}$. Note, however, that the latter separation is unlikely to be possible at experiments with high mean boost because then $\beta \simeq 1$ over much of the momentum range.

A constraint $A_{+-} \lesssim 0.5^\circ$ on the amplitude A_{+-} of time variations of the phase ϕ_{+-} with sidereal periodicity was announced at this conference [2]. Equation (15) shows that A_{+-} is given by

$$A_{+-} = \beta\gamma \frac{\sin\hat{\phi}\sin\chi}{|\eta_{+-}|\Delta m}\sqrt{(\Delta a_X)^2 + (\Delta a_X)^2} \quad . \tag{16}$$

Substitution for known quantities and for the experimental constraint on A_{+-} places the bound

$$\sqrt{(\Delta a_X)^2 + (\Delta a_Y)^2} \lesssim 10^{-20} \text{ GeV} \tag{17}$$

on the relevant parameters for CPT violation. Like the bound (11), the ratio of this bound to the kaon mass compares favorably with the ratio of the kaon mass to the Planck scale. Note that the bounds (11) and (17) represent independent constraints on possible CPT violation. Note also that in principle a constraint on the phase of the sidereal variations of ϕ_{+-}, determined by the ratio $\Delta a_Y/\Delta a_X$, would permit the separation of Δa_X and Δa_Y.

The examples discussed in this talk show that the study of momentum and time dependence in CPT observables is necessary to obtain the full CPT reach in a given experiment. Additional interesting results would emerge from careful analyses for experiments other than the ones considered here. Moreover, although emphasis has been given to the kaon system, related analyses in other neutral-meson systems would be well worth pursuing.

References

[1] E773 collaboration, B. Schwingenheuer *et al.*, Phys. Rev. Lett. **74** (1995) 4376. See also E731 collaboration, L.K. Gibbons et al., Phys. Rev. D **55** (1997) 6625; R. Carosi et al., Phys. Lett. B **237** (1990) 303.

[2] KTeV collaboration, presented by Y.B. Hsiung, these proceedings

[3] CPLEAR collaboration, presented by P. Bloch, these proceedings.

[4] P. Franzini, in G. Diambrini-Palazzi, C. Cosmelli, L. Zanello, eds., *Phenomenology of Unification from Present to Future,* World Scientific, Singapore, 1998; P. Franzini and J. Lee-Franzini, Nucl. Phys. Proc. Suppl. **71** (1999) 478.

[5] KLOE collaboration, presented by A. Antonelli, these proceedings; presented by S. Di Falco, these proceedings.

[6] See, for example, C. Bhat *et al.*, preprint FERMILAB-P-0894 (1998).

[7] OPAL Collaboration, R. Ackerstaff *et al.*, Z. Phys. C **76** (1997) 401.

[8] DELPHI Collaboration, M. Feindt *et al.*, preprint DELPHI 97-98 CONF 80 (1997).

[9] See, for example, T.D. Lee and C.S. Wu, Annu. Rev. Nucl. Sci. **16** (1966) 511.

[10] V.A. Kostelecký and S. Samuel, Phys. Rev. Lett. **63** (1989) 224; *ibid.*, **66** (1991) 1811; Phys. Rev. D **39** (1989) 683; *ibid.*, **40** (1989) 1886; V.A. Kostelecký and R. Potting, Nucl. Phys. B **359** (1991) 545; Phys. Lett. B **381** (1996) 89.

[11] V.A. Kostelecký and R. Potting, Phys. Rev. D **51** (1995) 3923.

[12] D. Colladay and V.A. Kostelecký, Phys. Rev. D **55** (1997) 6760; *ibid.*, **58** (1998) 116002.

[13] The discrete symmetries C, P, T and their combinations are discussed in, for example, R.G. Sachs, *The Physics of Time Reversal,* University of Chicago Press, Chicago, 1987.

[14] For a review, see B. Winstein and L. Wolfenstein, Rev. Mod. Phys. **65** (1993) 1113.

[15] See, for example, V.A. Kostelecký, ed., *CPT and Lorentz Symmetry,* World Scientific, Singapore, 1999.

[16] V.A. Kostelecký and R. Potting, in D.B. Cline, ed., *Gamma Ray–Neutrino Cosmology and Planck Scale Physics* (World Scientific, Singapore, 1993) (hep-th/9211116); D. Colladay and V.A. Kostelecký, Phys. Lett. B **344** (1995) 259; Phys. Rev. D **52** (1995) 6224; V.A. Kostelecký and R. Van Kooten, Phys. Rev. D **54** (1996) 5585.

[17] V.A. Kostelecký, Phys. Rev. Lett. **80** (1998) 1818; Phys. Rev. D **61** (2000) 16002.

[18] P.B. Schwinberg, R.S. Van Dyck, Jr., and H.G. Dehmelt, Phys. Lett. A **81** (1981) 119; Phys. Rev. D **34** (1986) 722; L.S. Brown and G. Gabrielse, Rev. Mod. Phys. **58** (1986) 233; R.S. Van Dyck, Jr., P.B. Schwinberg, and H.G. Dehmelt, Phys. Rev. Lett. **59** (1987) 26; G. Gabrielse *et al.*, *ibid.*, **74** (1995) 3544.

[19] R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. **79** (1997) 1432; Phys. Rev. D **57** (1998) 3932.

[20] G. Gabrielse *et al.*, in Ref. [15]; Phys. Rev. Lett. **82** (1999) 3198.

[21] H. Dehmelt *et al.*, Phys. Rev. Lett. **83** (1999) 4694.

[22] R. Mittleman, I. Ioannou, and H. Dehmelt, in Ref. [15]; R. Mittleman *et al.*, Phys. Rev. Lett. **83** (1999) 2116.

[23] S.M. Carroll, G.B. Field, and R. Jackiw, Phys. Rev. D **41** (1990) 1231.

[24] R. Jackiw and V.A. Kostelecký, Phys. Rev. Lett. **82** (1999) 3572.

[25] M. Pérez-Victoria, Phys. Rev. Lett. **83** (1999) 2518; J.M. Chung, Phys. Lett. B **461** (1999) 138.

[26] M. Charlton *et al.*, Phys. Rep. **241** (1994) 65; J. Eades, ed., *Antihydrogen,* J.C. Baltzer, Geneva, 1993.

[27] R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. **82** (1999) 2254.

[28] V.W. Hughes, H.G. Robinson, and V. Beltran-Lopez, Phys. Rev. Lett. **4** (1960) 342; R.W.P. Drever, Philos. Mag. **6** (1961) 683; J.D. Prestage *et al.*, Phys. Rev. Lett. **54** (1985) 2387; S.K. Lamoreaux *et al.*, Phys. Rev. Lett. **57** (1986) 3125; Phys. Rev. A **39** (1989) 1082; T.E. Chupp *et al.*, Phys. Rev. Lett. **63** (1989) 1541; C.J. Berglund *et al.*, Phys. Rev. Lett. **75** (1995) 1879.

[29] V.A. Kostelecký and C.D. Lane, Phys. Rev. D **60** (1999) 116010; J. Math. Phys. **40** (1999) 6245.

[30] R. Bluhm, V.A. Kostelecký and C.D. Lane, Phys. Rev. Lett., in press, hep-ph/9912451.

[31] S. Coleman and S. Glashow, Phys. Rev. D **59** (1999) 116008.

[32] O. Bertolami *et al.*, Phys. Lett. B **395** (1997) 178.

[33] The possibility that unconventional quantum mechanics in the kaon system might generate CPT violation is discussed in J. Ellis *et al.*, Phys. Rev. D **53** (1996) 3846, where it is shown that the resulting effects can be separated from δ_K.

[34] Direct CPT violation in the decay amplitudes is neglected in this talk because it is expected to be unobservable in the standard-model extension [11].

[35] *Review of Particle Properties,* Eur. Phys. J. C **3** (1998) 1.

[36] The possibility of a relatively large momentum dependence for T violation is considered in, for example, J.S. Bell and J.K. Perring, Phys. Rev. Lett. **13** (1964) 348; S.H. Aronson *et al.*, Phys. Rev. D **28** (1983) 495.

[37] J. Adams *et al.*, Phys. Rev. Lett. **79** (1997) 4093.

[38] T.T. Wu and C.N. Yang, Phys. Rev. Lett. **13** (1964) 380.

[39] V.V. Barmin et al., Nucl. Phys. **B247** (1984) 293.

[40] N.W. Tanner and R.H. Dalitz, Ann. Phys. **171** (1986) 463.

[41] KTeV collaboration, A. Alavi-Harati *et al.*, preprint EFI 99-25 (1999). See also NA31 collaboration, G.D. Barr *et al.*, Phys. Lett. B **317** (1993) 233; E731 collaboration, L.K. Gibbons *et al.*, Phys. Rev. Lett. **70** (1993) 1203.

[42] See, for example, C.D. Buchanan et al., Phys. Rev. D **45** (1992) 4088.